# AN ITERATIVE CONTROLLER DESIGN SCHEME BASED ON AVERAGE ROBUST CONTROL

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Abstract: In recent years, a great deal of attention has been devoted to iterative control as an efficient methodology for the design of highly-performing controllers. In this paper, we propose a new iterative scheme which explicitly accounts for the presence of uncertainty. At each step, the designed controller is the best possible one relative to the existing level of uncertainty and uncertainty is reduced through steps. In this way, the achieved performance rapidly improves from one step to the next, while preserving the robust stability of the closed-loop system. The controller design is performed at a low computational effort thanks to the use of randomized algorithms.

Keywords: Iterative methods, robustness, closed-loop identification, Monte Carlo method, randomized algorithms

## 1. INTRODUCTION

The problem of designing a highly-performing controller for a plant $P$ whose dynamics is unknown is considered. Let us assume that $P$ comes with an initial feedback controller associated with it. This controller, however, guarantees poor performance which does not meet the control specifications. In this context, a typical way to proceed is to first identify a model of the plant and then to design a new controller on the basis of the identified model. It has been shown in the literature that a one-step identification of the plant often results in a model that is unsuitable for controller design purposes (see (Gevers, 2000) and (Van den Hof and Schrama, 1995).) The reason for this is that it is a-priori difficult to select a suitable model class so as to achieve a sensible compromise between the model complexity and the number of available data points.
A different approach which has been studied in the last decade is to perform a sequence of identification and control design stages (*iterative control*.) In this way, the designer learns how to compromise between different needs through experience as time progresses (see (Lee *et al.*, 1993), (Van den Hof and Schrama, 1995) and (Gevers, 2000).)

**Iterative control**
The idea of iterative control can be formalized as follows. Let us assume that the plant is a SISO discrete-time linear system whose transfer function is $P(z)$. $P(z)$ is unknown and can possibly be very complex. Let $R_0(z)$ be the transfer function of the initial controller connected in feedback with the plant.
Before we proceed, we need some notations. For a generic feedback system with plant $G(z)$ and controller $R(z)$, let $J(R,G)$ denote the control cost. $J$ is such that $J(R,G) \geq 0 \ \forall R,G$, and the lower $J$ the better the performance. As we will see in the sequel, the evaluation of $J$ can be performed either analytically or by data processing. The objective of the control problem we deal with is to find $\overline{R}(z)$ such that $J(\overline{R},P) \leq c$; in other words, we seek a controller ensuring that the performance level is no worse than a given level $c$ when applied to the plant $P(z)$.
In general terms, an iterative procedure consists of the following steps:

0. an initial controller $R_0(z)$ connected in feedback with the plant is given. Set $i = 1$;
1. collect data in closed-loop and estimate a model $\widehat{P}_i(z)$;
2. design a new controller $R_i(z)$ based on $\widehat{P}_i(z)$ and connect it to the plant (see figure 1);
3. check the result:
   3.1 if $J(R_i, P) \leq c$, then stop.
   3.2 else, put $i = i + 1$ and go to step 1.

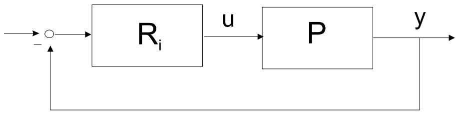


Figure 1. Closed-loop system.

The control design at Step 2 is performed via the analytical evaluation of $J$ based on the current estimated $\widehat{P}_i(z)$. On the contrary, the validation test of Step 3 is performed by means of data collected in closed loop while the real plant is operated with $R_i(z)$ as controller. As for Step 1, the benefits of adopting a closed loop identification procedure have been discussed in many papers, see e.g. (Van den Hof and Schrama, 1995).
At iteration $i$, the collected data will exhibit a frequency content that covers a range up to approximately the crossover frequency of the control system formed by $R_{i-1}(z)$ and $P(z)$. As a consequence, we cannot expect the identification at Step 1 to reveal all the dynamics of the plant and the next controller $R_i(z)$ will have to be designed on a conservative ground. In particular, it should have a limited bandwidth (cautious controller.)
In order to proceed cautiously, typically Step 2 is split into a number of sub-steps: $R_i(z)$ is not directly designed; rather, a sequence of intermediate controllers are selected, each one generating an increased bandwidth. After each design, the stability margin is monitored by implementing the new controller and inspecting the closed-loop system for possible oscillations (see (Lee *et al.*, 1993), (Anderson *et al.*, 1998) and (Bitmead *et al.*, 1997).) It should be noted that all the intermediate controllers used in the various phases of Step 1 are based on the same model $\widehat{P}_i(z)$. When oscillations are observed, the model $\widehat{P}_i(z)$ is no longer reliable and a new model has to be identified. This means that Step 2 is halted and the procedure moves on to Step 3.

**Robust iterative control**
The above outlined standard way of proceeding in iterative schemes has a drawback: each intermediate controller has to be tested on the real plant and this requires to stop the plant operation many times for experiments. This problem can be alleviated by resorting to a robust iterative control setting.
One first important observation is that, in the above procedure, only the nominal estimated model is used while there is no concern at all for model uncertainty. On the other hand, many identification techniques deliver an uncertainty region associated with the nominal model. The new method we propose in this paper relies on the exploitation of this extra information. This is obtained by replacing steps 1 and 2 with the following ones:

1′. collect data in closed-loop and estimate a model $\widehat{P}_i(z)$ along with its uncertainty;
2′. design the *best possible robust controller* $R_i(z)$ according to the existing level of uncertainty. Connect it to the plant;

The idea behind points 1′ and 2′ can be explained as follows. At iteration $i$, a sensible selection of the controller has to meet two different and contrasting objectives:

i) on the one hand, the controller has to be cautious to avoid a possible destabilization of the control system;

ii) on the other hand, it should not be over-cautious, otherwise the corresponding performance improvement is not significant.

The robust controller design in 2′ captures a compromise between the above two objectives. In one single step the best possible controller compatible with the present level of uncertainty is designed. This is contrast with Step 2, where neglecting uncertainty has the consequence of requiring the splitting of the step into a number of sub-steps, with corresponding experimental over-effort.
For another interesting contribution in iterative robust control, along a different line than the present work, see (De Callafon and Van den Hof, 1997).

**Structure of the paper**
In Section 2, we introduce our robust control approach. It is based on an average cost criterion. Randomized algorithms for the computation of the corresponding controller are presented in Section 3, leading to the complete robust iterative algorithm summarized in Section 4. Finally, a simulation example concludes the paper in Section 5.

## 2. AVERAGE ROBUST CONTROL

Let $\mathscr{P} = \left\{P(z,\vartheta) : \vartheta \in \Theta \subseteq \mathbb{R}^p\right\}$ be the parameterized set of feasible models, where $P(z,\vartheta)$ is a rational transfer function. Typically, $\vartheta$ is the vector of coefficients of the numerator and the denominator polynomials.
A prediction error identification procedure is used. This identification returns a nominal model, namely $P(z,\widehat{\vartheta})$ and a probability density $f : \Theta \to \mathbb{R}$, describing the likelihood that model $P(z,\vartheta)$ is the true system. Under certain assumptions, this probability density is in fact a Gaussian with the nominal model as mean and a variance which can be estimated from data (see (Ljung, 1987) and (Ljung, 1999).)
Suppose that a set of controllers $\mathscr{R}$ parameterized by a vector $\gamma$ is also given:

$$\mathscr{R} = \left\{ R(z,\gamma) : \gamma \in \Gamma \subseteq \mathbb{R}^q \right\}.$$

As a typical example, one can think of the PID class, in which case:

$$\mathscr{R} = \left\{ \frac{\gamma_1 + \gamma_2 z^{-1} + \gamma_3 z^{-2}}{1 - z^{-1}} : (\gamma_1, \gamma_2, \gamma_3) \in \mathbb{R}^3 \right\}.$$

The average cost criterion $c(\gamma)$ is defined as follows. Let $J(\gamma, \vartheta)$ be a shorthand for the control index $J(R(z,\gamma), P(z,\vartheta))$. Then,

$$c(\gamma) = \int_{\Theta} J(\gamma, \vartheta) f(\vartheta) \mathrm{d}\vartheta. \quad (1)$$

In this way, the performance index $J(\gamma, \vartheta)$ associate with each model $\vartheta$ is weighted according to the corresponding likelihood $f(\vartheta)$, so as to build the average performance $c(\gamma)$ of the controller $R(z,\gamma)$.
The optimal average robust controller is found by minimizing $c(\gamma)$:

$$\gamma^o = \arg\min_{\gamma \in \Gamma} c(\gamma) \quad (2)$$

*Remark 1.* To find the controller parameters, one could of course resort to *worst-case* robust control techniques as well. In our experience, however, average robust control performs better in iterative control applications. The reason is that the worst-case philosophy may result in over-conservative controllers and this slows down the performance improvement through iterations. □

*Remark 2.* The average cost criterion (1) can be minimized at a low computational effort by means of randomized algorithms (see e.g. (Campi and Prandini, to appear), (Campi and Prandini, 1998), and (Vidyasagar, 1997)). For the sake of completeness, in the next section we provide a short resume of the results useful in our context. □

## 3. RANDOMIZED ALGORITHMS

The randomized algorithms are Montecarlo-like methods that compute an approximation of the average robust controller (2), where the level of approximation can be specified a-priori.
Let $\{\gamma_1, \ldots, \gamma_n\}$ be $n$ controller parameters selected in such a way that they densely cover the feasible set $\Gamma$. We search for the best controller parameter among $\{\gamma_1, \ldots, \gamma_n\}$, rather than over the entire feasible set $\Gamma$.
In order to compute

$$\overline{\gamma}^o = \arg \min_{\gamma \in \{\gamma_1, \ldots, \gamma_n\}} c(\gamma),$$

an empirical counterpart of the average cost (1) is used. Precisely, define

$$\widehat{c}(\gamma) = \frac{1}{m} \sum_{k=1}^{m} J(\gamma, \vartheta_k),$$

where, $\vartheta_k$'s are $m$ vectors independently extracted from $\Theta$ according to the probability density $f$, and let

$$\overline{\gamma} = \arg \min_{\gamma \in \{\gamma_1, \ldots, \gamma_n\}} \widehat{c}(\gamma).$$

It is possible that $\overline{\gamma} \neq \overline{\gamma}^o$. However, by a suitable selection of $m$, the difference $c(\overline{\gamma}) - c(\overline{\gamma}^o)$ can be made arbitrarily small. This is precisely stated in the following theorem (see (Vidyasagar, 1997) and also (Campi and Prandini, to appear) and (Campi and Prandini, 1998).)

*Theorem 1.* :

Fix two real numbers $\varepsilon > 0$ and $\delta > 0$ and assume that $J(\gamma, \vartheta) \in [0,1]$, $\forall \gamma, \theta$.
If $m > (2\varepsilon^2)^{-1} \ln(2n/\delta)$ then, $c(\overline{\gamma}) \leq c(\overline{\gamma}^o) + 2\varepsilon$ with a probability greater than or equal to $1 - \delta$. □

*Remark 3.* Note that $m$ does not depend on the size $p$ of the space in which $\Theta$ is embedded. This is in contrast with standard non-random numerical methods for computing integrals. □

*Remark 4.* The condition $J(\gamma, \vartheta) \in [0,1]$ can in general be fullfilled by a suitable re-scaling of the control cost. □

Before proceeding, we are well advised to raise a delicate point; namely the curse of dimensionality. Indeed, in order to explore the entire controller set, the integer $n$ must increase exponentially with $q$, the dimensionality of the controller parameter space, so that it becomes very large even for relatively small values of $q$. Correspondingly, the computational burden of the algorithm for the search of the best controller becomes rapidly intractable. One point of strength of the method we will propose in Section 5 is that $q = 1$ there, so that this problem automatically disappears.

## 4. COMPLETE ITERATIVE ALGORITHM

By complementing the algorithm of Section 1 with the randomized methods discussed in the previous section, we arrive to the following iterative robust algorithm:

0. an initial controller $R_0(z)$ connected in feedback with the plant is given.
   Choose the model class $\mathscr{P}$ and the controller class $\mathscr{R}$. Sample $\Gamma$ with $\{\gamma_1, \ldots, \gamma_n\}$.
   Select $\varepsilon$ and $\delta$ and let $m > \frac{1}{2\varepsilon^2} \ln \frac{2n}{\delta}$.
   Set $i = 1$;
1. collect data in closed-loop and estimate a model $\widehat{P}_i(z) = P(z, \widehat{\vartheta}_i)$ along with the density $f_i$;
2. extract $\vartheta_k^i$, $k = 1 \ldots m$ according to $f_i$.
   Let

$$\overline{\gamma}_i = \arg \min_{\gamma \in \{\gamma_1, \ldots, \gamma_n\}} \frac{1}{m} \sum_{k=1}^{m} J(\gamma, \vartheta_k^i)$$

and set $R_i(z) = R(z, \overline{\gamma}_i)$.
Connect $R_i(z)$ to the plant;
3. check the result:
3.1 if $J(R_i, P) \leq c$, then stop.
3.2 else, put $i = i+1$ and go to step 1.

## 5. APPLICATION EXAMPLE: THE GRENOBLE FLEXIBLE TRANSMISSION SYSTEM

In this section, an application of the average robust iterative algorithm presented in Section 4 is described. The presented example has been chosen for its simplicity in order to focus on some issues of this new iterative control design rather than on technical details. In fact, many implementation features discussed herein are of general interest.

**Statement of the example**
We consider the Grenoble transmission system presented in (Landau *et al.*, 1995). The system is constituted by three pulleys connected by two elastic belts as shown in Figure 2, and its transfer function is given by:

$$P(z) = \frac{0.033z + 0.054}{z^4 - 2.85z^3 + 3.72z^2 - 2.65z + 0.87}.$$

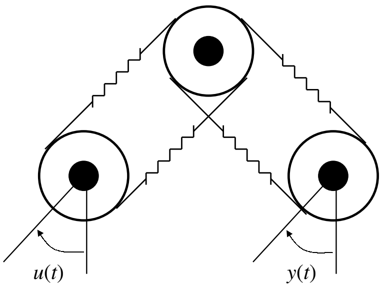


Figure 2. The Grenoble transmission system.

Such transfer function is characterized by two couples of complex conjugate stable poles, giving rise to two resonant peaks (see Figure 3.) One zero outside the unit circle is also present.

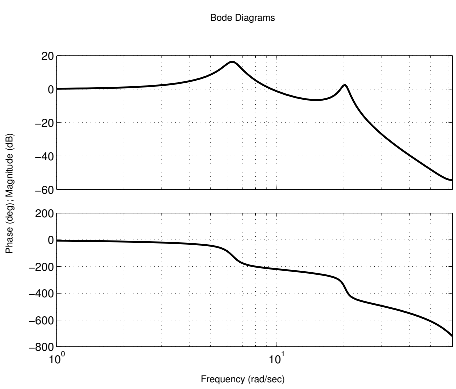


Figure 3. Bode diagram of $P(z)$.

In the simulations, the system output is corrupted by additive white noise $d(t)$ having zero mean and variance equal to 0.01 , namely:

$$y(t) = P(z)u(t) + d(t).$$

We suppose that the system is initially connected with a conservative controller which results in a stable but very slow closed loop system.

**Determining $f_i$**
For identification, the FIR model class

$$y(t) = P(z, \vartheta)u(t) + \xi(t)$$

has been used, where

$$P(z, \vartheta) = \vartheta_1 z^{-1} + \vartheta_2 z^{-2} + \ldots + \vartheta_{50} z^{-50}$$

and $\xi(t) \sim \mathrm{WN}(0, \lambda^2)$.
Using 50 parameters is advisable in order to capture the entire dynamics of the system. As we have already noticed, considering models with many parameters does not adversely affect the randomized algorithm since the computational burden is not influenced by the dimensionality of $\Theta$ (see Remark 3.) Notice also that in this example the true system structure is simple enough and one could directly estimate its parameters. However, we need to keep in mind that the plant structure can not be considered to be known in real application.
The closed loop identification of the FIR model is easily performed directly (i.e. by measuring the plant $P$ input and output signals) by means of *least squares* techniques, along standard lines (Ljung, 1987). The density $f_i$ of the estimated model is evaluated by resorting to the asymptotic theory.

**Controller class**
The controller class is updated through iterations according to the following rationale.
At iteration $i$, an additional model $\widehat{M}_i(z) = \frac{\widehat{B}_i(z)}{\widehat{A}_i(z)}$ of reduced complexity is first identified (an ARMAX(4,2,4) has been considered). $\widehat{M}_i(z)$ is used to design a deadbeat controller $\widehat{R}_i(z)$:

$$\widehat{R}_i(z) = \frac{\widehat{A}_i(z)}{\widehat{B}_i(1)z^p - \widehat{B}_i(z)}$$

which corresponds to the complementary sensitivity function:

$$\widehat{F}_i(z) = \frac{\widehat{B}_i(z)}{\widehat{B}_i(1)z^p}.$$

It should be noted that $\widehat{M}_i(z)$ has nothing to do with $P(z, \widehat{\vartheta}_i)$. The reason for identifying this additional reduced complexity model $\widehat{M}_i(z)$ (rather than using the nominal model $P(z, \widehat{\vartheta}_i)$) is to generate a deadbeat controller of simple structure.
The controller class is then defined as

$$R_i(z, \gamma) = H(z, \gamma)\widehat{R}_i(z).$$

where $H(z,\gamma)$ is a *detuning filter* which is introduced to decrease the crossover frequency of the control system. In this way, robustness in the control system is incorporated.
In the present application, a simple proportional action has been used as detuning filter: $H(z,\gamma) = \gamma$, with $\gamma \in [0,1]$. Figure 4 shows its effect on the nominal open-loop transfer function.

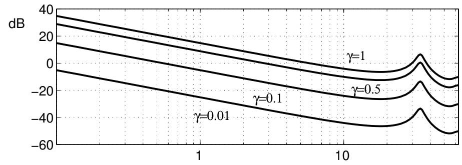


Figure 4. Bode diagram of $\gamma\widehat{R}_i(z)\widehat{M}_i(z)$ as a function of $\gamma$.

In general, the choice of the detuning filter is an open problem, currently underway.

**Cost criterion**
The cost criterion is:

$$J_i(\gamma,\vartheta) = \begin{cases} 1, & \text{if } (\gamma,\vartheta) \text{ is unstable} \\ 0.5\dfrac{\widetilde{J}_i(\gamma,\vartheta)}{1+\widetilde{J}_i(\gamma,\vartheta)}, & \text{otherwise} \end{cases}$$

where $(\gamma,\vartheta)$ denotes the closed-loop system of $R_i(z,\gamma)$ and $P(z,\vartheta)$ and

$$\widetilde{J}_i(\gamma,\vartheta) = \left\| \frac{R_i(\gamma)P(\vartheta)}{1+R_i(\gamma)P(\vartheta)} - \widehat{F}_i \right\|_2 .$$

Note that $J$ takes value in $[0,1]$.

**Simulation results**
Figure 5 represents the reduced order model $\widehat{M}_i(z)$ for $i = 1, 2, 3$.

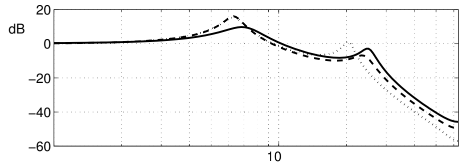


Figure 5. Estimated nominal models at the first three iteration ($i = 1$ ("—"), 2 ("- -"), 3 ("···").)

As for $f_i$, Figures 6 - 8 represent the Bode plot of some models extracted according to $f_1$, $f_2$, and $f_3$, respectively.

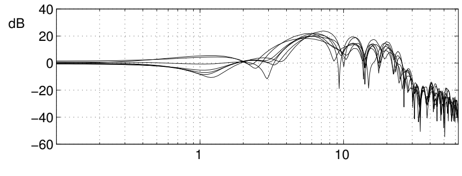


Figure 6. Uncertainty at the first iteration ($i = 1$.)

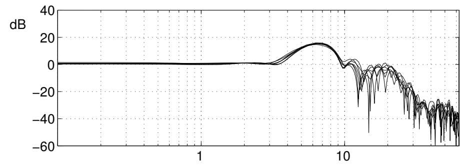


Figure 7. Uncertainty at the second iteration ($i = 2$.)

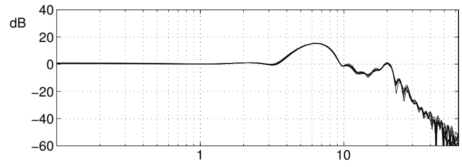


Figure 8. Uncertainty at the third iteration ($i = 3$.)

At the first iteration, $i = 1$, uncertainty is very scattered around the nominal model. The randomized algorithm has been applied with $\varepsilon = 0.1$ and $\delta = 0.1$. The control parameter has been selected by sampling the $[0,1]$ interval with a step equals to 0.025, leading to $n = 40$ and $m = 335$.
The resulting $\overline{\gamma}_1$ is equal to 0.075. Its small value indicates a conservative choice which is justified by the high level of uncertainty (see Figure 6.) The step-response of the closed-loop system at iteration 1 is depicted in Figure 9.

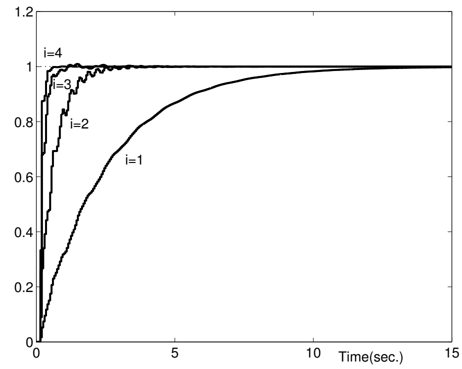


Figure 9. Step response of the closed loop at the first three iteration.

Carrying on the iterative procedure leads to the selection of $\overline{\gamma}_i$ as indicated in Figure 10; see Figure 9 for the corresponding closed loop step responses. Figure 11

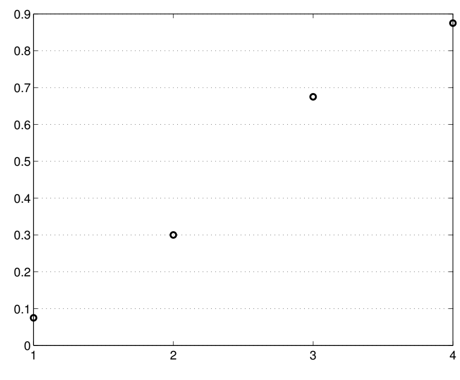

Figure 10. $\overline{\gamma}_i$ at each iteration.

represents the value of the empirical average cost $\widehat{c}(\overline{\gamma}_i)$ through iterations.

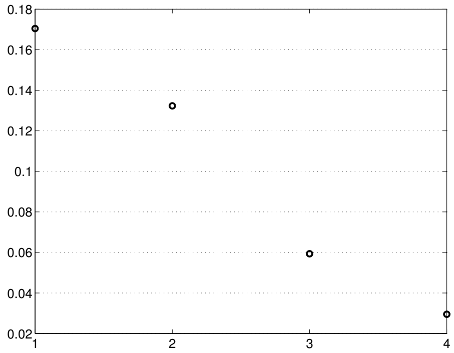


Figure 11. $\widehat{c}(\overline{\gamma}_i)$ plotted for each iterations.

*Remark 5.* One should note that the identification procedure is made simple in this example by the (unrealistic) assumption that the plant noise is white. Under general assumption, IV identification or identification of ARMA models can be used, while the general philosophy of the method remains unchanged. □

## 6. ACKNOWLEDGEMENT

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