SOME CRITICAL IMPLEMENTATION ISSUES IN ITERATIVE ROBUST CONTROL DESIGN

S. Bittanti * **M.C. Campi** ** **S. Garatti** *

* *Dipartimento di Elettronica ed Informazione - Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milano, Italy - {bittanti,sgaratti}@elet.polimi.it*
** *Dipartimento di Elettronica per l'Automazione - University of Brescia,Via Branze 38, 25123 Brescia, Italy - campi@ing.unibs.it*

Abstract: Iterative control is an efficient methodology for the design of highly-performing controllers. In this paper, we discuss many implementation issues of a new iterative scheme which explicitly accounts for the presence of uncertainty. The developed iterative enables one to improve quickly the performance through subsequent steps, while preserving the robust stability of the closed-loop system.

Keywords: Iterative methods, robustness, closed-loop identification, Montecarlo method, randomized algorithms

1. INTRODUCTION

The standard paradigm in control design is to work out a suitable controller once a model of the plant is given. When the model cannot be derived from physical considerations, one typically resorts to identification methods.
In the context of system identification, the one-shot identification strategy (i.e. the model is first identified and the controller is then designed based on the obtained model) may suffer from drawbacks in case of high levels of uncertainty, especially for the difficulties inherent in the a-priori selection of a model class (see (Gevers, 2000) and (Van den Hof and Schrama, 1995)).
In recent years, a great deal of attention has been devoted to iterative (identification and control) techniques. Here, the design is performed with a sequence of intertwined closed-loop identification and control steps, so as to progressively bring into light the plant dynamics and correspondingly improve the control system performance (see (Gevers, 2000), (Lee *et al.*, 1995) and (Van den Hof and Schrama, 1995)).

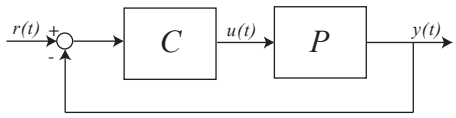

Fig. 1. Closed-loop system.

In the sequel, we will consider SISO discrete-time plants and we will adopt the following notation:

- $P(z)$ is the plant transfer function;
- i is the iteration index; each iteration is subdivided in two steps:
  $i.a$ identification of a model $\widehat{P}_i(z)$ of $P(z)$;
  $i.b$ design of a controller $C_i(z)$;
- $J(C,P)$ is the control cost for the pair formed by the controller $C(z)$ and plant $P(z)$ (Figure 1). We assume that $J(C,P) \geq 0$, $\forall C,P$, and that the lower J the better the performance. The objective of the control problem we deal with is to find a controller such that $J(\overline{C},P) \leq k$.

There is a variety of iterative techniques with different and specific features ((Lee *et al.*, 1995), (Van den Hof and Schrama, 1995), (Gevers, 2000), (De Callafon and

Van den Hof, 1997)). In the well known "windsurfer" approach ((Lee *et al.*, 1995)) step $i.b$ is split into a number of sub-steps: namely, starting from the model identified at step $i.a$, the controller is progressively tuned so as to enlarge the closed-loop bandwidth and at any sub-steps the designed controller is tested on the real plant to avoid reaching the instability limit. In principle, this is a wise cautious procedure since in the first iterations the identified model is still rather unreliable. When it is likely that a further increment in the bandwidth may lead to instability, one proceeds to the new iteration $i+1$ so as to improve the knowledge on the plant dynamics thanks to a new identification phase exploiting the data measured from the closed-loop system consisting of $\{C_i(z), P(z)\}$, where $C_i(z)$ is the last controller obtained at the end of the previous iteration. Unfortunately, this way of proceeding may require a number of intermediate steps and a relatively long design phase with many experiments on the real plant. The cautiousness of this approach is a feature shared by other standard iterative schemes (see (Anderson *et al.*, 1998) and (Bitmead *et al.*, 1997)).
As discussed in (De Callafon and Van den Hof, 1997) and (Bittanti *et al.*, 2002), a way of alleviating the drawbacks stemming from the cautiousness of the above approaches is to resort to *robust* iterative control techniques, i.e. iterative schemes which explicitly account for the presence of uncertainty in the control design phase. Along this robust line, point $i.a$ is split in two sub-points, and the whole identification-control procedure becomes:

$i.a$ from the data collected in closed-loop
  $i.a.1$ estimate the nominal model;
  $i.a.2$ estimate the model uncertainty;
$i.b$ design the *best possible robust controller* $C_i(z)$ according to the existing level of uncertainty. Connect it to the plant as in Figure 1;
$i.c$ check the result:
  $i.c.1$ if $J(C_i, P) \leq k$, then stop.
  $i.c.2$ else, put $i = i+1$ and go to step $i.a$.

The algorithm is initialized by connecting the plant with an initial controller $C_0(z)$. Typically, due to the uncertainty in the model plant, the performance of such initial control system is poor.
The idea behind points $i.a$ and $i.b$ above can be explained as follows. At iteration i, a sensible selection of the controller has to meet two different and contrasting objectives:

- on the one hand, the controller has to be cautious to avoid a possible destabilization of the control system;
- on the other hand, it should not be overconservative, otherwise the corresponding performance improvement is not significant.

The robust controller design performs in a single step the best compromise between the above two objectives according to the present level of uncertainty. In this way, the achieved performance rapidly improves from one iteration to the next, while preserving the robust stability of the closed-loop system. This is contrast with standard iterative schemes, where neglecting uncertainty has the consequence of requiring the splitting of the control design step into a number of sub-steps, with corresponding experimental over-effort.

The iterative algorithm outlined above describes the essential idea of iterative robust control at a general level. However, to fill the gap between the general idea and the real application, one has to decide how to perform the identification and the model quality estimation (step $i.a$), and also which kind of robust control method has to be used (step $i.b$).
These points do not seem to be well clarified in the existing literature, though it is apparent that addressing them is of paramount importance for the success of the iterative algorithm. The main goal of this paper is to explicitly discuss the above issues so as to provide a complete iterative robust control scheme.
Precisely, the estimation of the nominal model and model uncertainty in step $i.a$ is discussed in Section 2, while in Section 3 our robust control approach, based on an average cost criterion, is presented. Then, the complete iterative scheme is presented in Section 4, and a simulation example is finally given in Section 5.

2. THE IDENTIFICATION AND UNCERTAINTY ESTIMATION STEP

For the identification of the nominal model (step $i.a.1$) we consider standard Prediction Error Methods (PEM) as well as Instrumental Variable (IV) identification (see (Ljung, 1999)). The uncertainty evaluation is performed by means of the corresponding asymptotic theory. As it is well known ((Ljung, 1999)), in the asymptotic theory the uncertainty is assessed through a probability density $f_i(\vartheta)$ ($f_i : \Theta \to \mathbb{R}$, where $\vartheta \in \Theta \subseteq \mathbb{R}^n$ is a vector parameterizing the model class) describing the likelihood that the model corresponding to ϑ is the true system. Under weak assumptions, this probability density is Gaussian with mean and variance which can be estimated from the available data.
Although all these methods are well known in the literature, in the present framework some care is needed.

First of all, let us notice that for the success of the iterative procedure it is usually advisable to consider different classes of models for the two steps $i.a.1$ and $i.a.2$. The reason for this lies in the fact that the nominal model and the uncertainty description play different roles in the subsequent control design step $i.b$ (see also Section 3 where step $i.b$ is discussed). Indeed, the nominal model is used to design a nominal controller and therefore a class of low order models is advisable to obtained low complexity controllers. The uncertainty description, instead, is used to detune the previous controller parameters so as to meet a robust stability requirement. It is clear that in this second phase it is important to use a full order model class so as to capture all the uncertainty in the nominal model.

In the sequel, we will denote by $\mathscr{M}_\lambda$, $\lambda \in \Lambda \subset \mathbb{R}^m$, the parametric class of low order models, whereas $M(z,\widehat{\lambda}_i)$ will denote the nominal model identified at iteration i. Finally, $\mathscr{P}_\vartheta$, $\vartheta \in \Theta \subset \mathbb{R}^n$, will be the full-order model class used for model quality assessment.

Before proceeding, one should be also aware that the choice of the model class $\mathscr{P}_\vartheta$ is critical since, for certain classes of models and in condition of poor excitation, the asymptotic theory of system identification may lead to completely unreliable results. Indeed, as shown in (Garatti *et al.*, 2004), the estimated density may be extremely peaky so suggesting that uncertainty is restricted, and nevertheless the real plant dynamics is located far from the peak. Note that poor excitation conditions are likely to occur in iterative control since, during the first iterations, the controller is cautious and therefore the bandwidth is limited. According to the discussion in (Garatti *et al.*, 2004) and (Garatti *et al.*, 2003), the above problem can be avoided using IV identification. When resorting to PEM methods, then the class of models $\mathscr{P}_\vartheta$ has to be carefully chosen, see (Garatti *et al.*, 2004) and (Garatti *et al.*, 2003) for motivations and more detailed explanation.

3. THE ROBUST CONTROLLER DESIGN STEP

The objective of this section is to describe how the information supplied by step *i.a* (i.e. $M(z,\widehat{\lambda}_i)$ and $f_i(\vartheta)$) can be used to design the "best possible" robust controller $C_i(z)$ in step *i.b*.
Here, we consider a two-steps design method:

1. Design a *nominal controller* $\widehat{C}_i(z)$ based on the nominal model $M(z,\widehat{\lambda}_i)$.
2. *Detune* the nominal controller parameters according to the existing level of uncertainty so as to meet a robust stability requirement.

These two points are now discussed in turn.
The nominal controller is typically obtained by minimizing the control cost $J(C(z),M(z,\widehat{\lambda}_i))$ with respect to $C(z)$. The result of nominal controller design is usually a high performing controller, which, however, is also very sensitive to model inaccuracy.
The detuning of the nominal controller is obtained through a detuning filter $H(z,\gamma)$, $\gamma \in \Gamma \subseteq \mathbb{R}^l$, which has to be used in connection with $\widehat{C}_i(z)$, so obtaining the controller $C_i(z) = H(z,\gamma)\cdot\widehat{C}_i(z)$. By varying γ, it is possible to incorporate robustness in the nominal controller so as to make $C_i(z)$ suited to stabilize the plant, even though the latter is different from the nominal model. The price of the detuning is typically a degradation of the nominal performance.
As is obvious the value of γ for the current iteration has to be chosen according to the existing level of uncertainty as described by $f_i(\vartheta)$. In this work,we propose an average robust approach for such a selection.
Suppose that the control cost $J(C,P)$ takes on quite large values when the pair (C,P) generates an unstable closed-loop system (i.e. the instability is penalized by $J(C,P)$). Then, the average cost criterion $c(\gamma)$ is built as follows:

$$c(\gamma) = \int_\Theta J(\gamma,\vartheta)\, f_i(\vartheta)\mathrm{d}\vartheta = \mathbb{E}_\Theta[J(\gamma,\vartheta)],$$

where $J(\gamma,\vartheta)$ is a shorthand for $J(C(z,\gamma),P(z,\vartheta))$. In this way, the performance index $J(\gamma,\vartheta)$ is weighted according to the likelihood of ϑ given by $f_i(\vartheta)$, so that $c(\gamma)$ measures the average performance of $C(z,\gamma)$ for the existing uncertainty.
The optimal average robust controller $C(z,\gamma^o)$ is then found by minimizing $c(\gamma)$, i.e.

$$\gamma^o = \arg\min_{\gamma\in\Gamma} c(\gamma). \tag{1}$$

Remark 1. To find the controller parameters, one could of course resort to *worst-case* robust control methods as well. In our experience, however, average robust control performs better in iterative control applications. The reason is that the worst-case philosophy may result in over-conservative controllers and this slow down the performance improvement through iterations.

The average robust controller 1 can be computed at a low effort by means of the so-called randomized algorithms (see e.g. (Vidyasagar, 2001) and (Campi and Prandini, 2003)). For the sake of completeness, a short resume of the results useful in the iterative control context is provided in the following.

Randomized algorithms
The randomized algorithms are Montecarlo-like methods that compute an approximation of the average robust controller 1, where the level of approximation can be a-priori specified.
Let $\{\gamma_1,\ldots,\gamma_p\}$ be p samples of Γ. We search for the best controller parameter among $\{\gamma_1,\ldots,\gamma_p\}$, rather than over the entire feasible set Γ. In other words,

$$\overline{\gamma}^o = \arg\min_{\gamma\in\{\gamma_1,\ldots,\gamma_p\}} c(\gamma),$$

is computed in place of γ^o. We suppose that the samples $\{\gamma_1,\ldots,\gamma_p\}$ are drawn in such a way to densely cover the feasible set Γ.
In order to compute $\overline{\gamma}^o$, an empirical counterpart of the average cost $c(\gamma)$ is used. Precisely, define

$$\widehat{c}(\gamma) = \frac{1}{q}\sum_{k=1}^{q} J(\gamma,\vartheta_k),$$

where ϑ_k's are q parameter vectors independently extracted from Θ according to the probability density $f_i(\vartheta)$, and let

$$\overline{\gamma} = \arg\min_{\gamma\in\{\gamma_1,\ldots,\gamma_p\}} \widehat{c}(\gamma).$$

As it is obvious $\overline{\gamma} \neq \overline{\gamma}^o$ in general. Nevertheless, the difference $c(\overline{\gamma}^o) - c(\overline{\gamma})$ (i.e. the difference between the ideal average optimal performance and the actual one) can be made arbitrarily small by a suitable selection of q, as precisely stated in the following theorem (see (Vidyasagar, 2001) and (Campi and Prandini, 2003)).

Theorem 1. Fix two real numbers $\varepsilon > 0$ and $\delta > 0$ and assume that $J(\gamma, \vartheta) \in [0,1]$, $\forall \gamma, \theta$.
If $q > (2\varepsilon^2)^{-1} \ln(2p/\delta)$ then, $c(\overline{\gamma}) \leq c(\overline{\gamma}^o) + 2\varepsilon$ with a probability greater than or equal to $1-\delta$.

Remark 2. The condition $J(\gamma, \vartheta) \in [0,1]$ can in general be fulfilled by suitably re-scaling the control cost.

Remark 3. Note that, in contrast to standard non-random numerical method, q does not depend either on the smoothness of $J(\gamma, \vartheta)$ or on the size n of the space in which Θ is embedded. This allows in general to keep the computational effort of randomized algorithms small.

Remark 4. Before proceeding, one should be also aware of the fact that, in order to explore the entire controller set, p, the number of samples $\{\gamma_1, \ldots, \gamma_p\}$, must increase exponentially with l, the dimensionality of the controller parameter space. In this way, p becomes very large even for relatively small values of l (*curse of dimensionality*) and correspondingly, the computational burden of the algorithm for the search of the best controller becomes rapidly intractable. However, in contrast to what happened for Θ, the dimensionality of Γ is not required to be large. Rather, as we will see in Section 5, in many cases $l = 1$ suffices, so that this problem automatically disappears.

4. A COMPLETE ITERATIVE ROBUST CONTROLLER DESIGN SCHEME

By complementing the algorithm described in Section (1) with all the points discussed in previous sections, we obtain the following iterative robust algorithm.

Step 0: an initial controller $C_0(z)$ is connected in feedback with the plant. Choose the model class $\mathscr{M}_\lambda$ along with the model class $\mathscr{P}_\vartheta$. Choose also the detuning filter $H(z,\gamma)$, $\gamma \in \Gamma$. Sample Γ with $\{\gamma_1, \ldots, \gamma_p\}$. Select ε and δ. Let $q > \frac{1}{2\varepsilon^2} \ln \frac{2p}{\delta}$. Set $i = 1$;

i.a from the data collected in closed-loop
- *i.a*.1 identify a low-order model $M(z, \widehat{\lambda}_i)$ in $\mathscr{M}_\lambda$;
- *i.a*.2 estimate the probability density $f_i(\vartheta)$ over the high order class of models $\mathscr{P}_\vartheta$;

i.b design $\widehat{C}_i(z)$ based on $M(z, \widehat{\lambda}_i)$. Extract ϑ_k^i, $k = 1 \ldots q$ according to $f_i(\vartheta)$ and let

$$\overline{\gamma}_i = \arg \min_{\gamma \in \{\gamma_1, \ldots, \gamma_p\}} \frac{1}{q} \sum_{k=1}^{q} J(\gamma, \vartheta_k^i).$$

Set $C_i(z) = H(z, \overline{\gamma}_i)\widehat{C}_i(z)$. Connect it to the plant;

i.c check the result:
- i.c.1 if $J(C_i, P) \leq k$, then stop.
- i.c.2 else, put $i = i + 1$ and go to step *i.a*.

5. APPLICATION EXAMPLE

In this section an application example of the iterative algorithm outlined in Section 4 is presented. The presented example has been chosen for its simplicity in order to focus on some issues of the new iterative controller design schemes rather than on technical details. Many implementation features discussed herein are of general interest.

The plant description
We consider the well known Grenoble transmission system presented in (Landau *et al.*, 1995). This system is constituted by three pulleys connected by two elastic belts as shown in Figure 2, and its transfer function

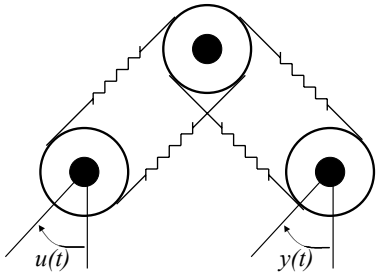

Fig. 2. The Grenoble transmission system.

is given by:

$$P(z) = \frac{0.033z + 0.054}{z^4 - 2.85z^3 + 3.72z^2 - 2.65z + 0.87}.$$

Such transfer function is characterized by two pairs of complex conjugate stable poles, giving rise to two resonant peaks. A zero outside the unit circle (non minimum phase system) is also present.
In the simulations, the system output is corrupted by an additive noise $d(t)$. Namely:

$$y(t) = P(z)u(t) + d(t), \tag{2}$$

where

$$d(t) = \frac{z-2}{z-0.9} e(t), \quad e(t) = WGN(0, 0.0025)$$

(WGN = White Gaussian Noise). Note that $d(t)$ is a high-correlated stochastic noise as it is typical of many real applications. Moreover, the noise level is quite high (the variance of $d(t)$ turns out to be 0.02).
The system is initially connected with the controller

$$C_0(z) = 0.05 \cdot \frac{z^4 - 2.85z^3 + 3.72z^2 - 2.65z + 0.87}{0.08z^4 - 0.03z - 0.05},$$

which results in a stable but slow closed-loop system.

Identification and uncertainty estimation
The nominal model $M(z, \widehat{\lambda}_i)$ of reduced complexity is identified, at each iteration i, through the following class of ARMAX(4,2,4) models:

$$\mathscr{M}_\lambda = \left\{ y(t) = \frac{B(z,\lambda)}{A(z,\lambda)} u(t) + \frac{C(z,\lambda)}{A(z,\lambda)} \eta(t) \right\},$$

where $\eta(t) = WN(0, \sigma^2)$ and λ is the vector of A, B, C coefficients.

As for the estimation of $f_i(\vartheta)$, two different high-order model class $\mathscr{P}_\vartheta^1$ and $\mathscr{P}_\vartheta^2$ are considered.
The first is a full-order Box-Jenkins model class:

$$\mathscr{P}_\vartheta^1 = \left\{ y(t) = G(z,\vartheta)u(t) + H(z,\vartheta)\xi(t) \right\},$$

where $\xi(t) = WN(0,\mu^2)$ and ϑ is the vector of the numerator and denominator polynomial coefficients of G and H. The probability density $f_i^1(\vartheta)$ is evaluated by resorting to the asymptotic theory of Prediction Error Methods.

According to (Garatti *et al.*, 2004) the evaluation of the uncertainty can be unreliable for Box-Jenkins model classes. $\mathscr{P}_\vartheta^1$ has been introduced here to merely show that the reliability problem of the asymptotic theory can be severe in iterative control, if $\mathscr{P}_\vartheta$ is not chosen with care (see Section 2).

The second model class which has been used to estimate the model uncertainty is as follows:

$$\mathscr{P}_\vartheta^2 = \Big\{ y(t) = P(z,\vartheta)u(t) + v(t) \Big\},$$

where $v(t)$ is a noise process and $P(z,\vartheta)$ is parameterized through Finite Impulse Response (FIR) filters, i.e. $P(z,\vartheta) = \vartheta_1 z^{-1} + \vartheta_2 z^{-2} + \ldots + \vartheta_n z^{-n}$.

In this case, the identification is performed through the Instrumental Variable method, and the probability density $f_i^2(\vartheta)$ is evaluated by resorting to the asymptotic theory of IV techniques as well.

$\mathscr{P}_\vartheta^2$ is advisable in the iterative robust control for the following reasons:

1. The asymptotic theory does not suffer from problems of reliability in this case ((Garatti *et al.*, 2003)).
2. High order FIR models are well suited to provide a full description of the true plant since the number n of parameters necessary to describe $P(z)$ can be determined in real applications by simply inspecting the impulse plant response.

For the Grenoble transmission system, a FIR model with 50 parameters has been selected in order to capture the entire dynamics of the plant. As we have already noticed, considering models with many parameters does not adversely affect the randomized algorithms (see Section 3).

Nominal controller and detuning

The nominal controller $\widehat{C}_i(z)$ of simple structure is obtained through the nominal identified model $M(z,\widehat{\lambda}_i) = B(z,\widehat{\lambda}_i)/A(z,\widehat{\lambda}_i)$ according to the deadbeat control method:

$$\widehat{C}_i(z) = \frac{A(z,\widehat{\lambda}_i)}{B(1,\widehat{\lambda}_i)z^k - B(z,\widehat{\lambda}_i)},$$

where k is equal to the order of $A(z,\widehat{\lambda}_i)$. $\widehat{C}_i(z)$ leads to the following complementary sensitivity function when it is connected with $M(z,\widehat{\lambda}_i)$:

$$\widehat{F}_i(z) = \frac{B_i(z,\widehat{\lambda}_i)}{B_i(1,\widehat{\lambda}_i)z^k}.$$

As a detuning filter $H(z,\gamma)$, a simple proportional action has been used $H(z,\gamma) = \gamma$, $\gamma \in [0,1]$.

The idea is that through γ it is possible to decrease the crossover frequency of the loop function $\gamma\widehat{C}_i(z)P(z)$, so that the controller robustness is increased.

The cost criterion

The cost criterion is:

$$J_i(\gamma,\vartheta) = \begin{cases} 1, & \text{if } (\gamma,\vartheta) \text{ is unstable} \\ 0.5\dfrac{\widetilde{J}_i(\gamma,\vartheta)}{1+\widetilde{J}_i(\gamma,\vartheta)}, & \text{otherwise} \end{cases}$$

where (γ,ϑ) denotes the closed-loop system of $R_i(z,\gamma)$ and $P(z,\vartheta)$ and

$$\widetilde{J}_i(\gamma,\vartheta) = \left\| \frac{C_i(\gamma)P(\vartheta)}{1+C_i(\gamma)P(\vartheta)} - \widehat{F}_i \right\|_2.$$

Note that J takes value in $[0,1]$.

Simulation results

The iterative robust controller design scheme in Section 4 has been applied to the present example.

Figure 3 represents the reduced order model $M(z,\widehat{\lambda}_1)$ estimated at the first iteration ($i = 1$). As for the estimated probability density, Figure 4 and 5 represent the Bode plot of some models extracted according to $f_1^1(\vartheta)$ (Box-Jenkins model class) and $f_1^2(\vartheta)$ (FIR model class).

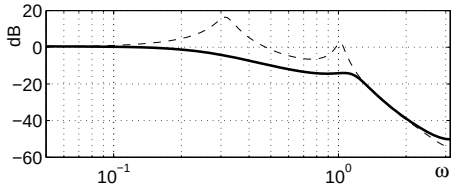

Fig. 3. Estimated nominal model at the first iteration ("—") and true system ("- -") bode diagrams.

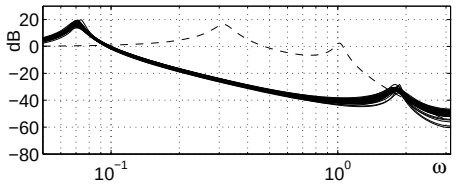

Fig. 4. Uncertainty at the first iteration (Box-Jenkins models).

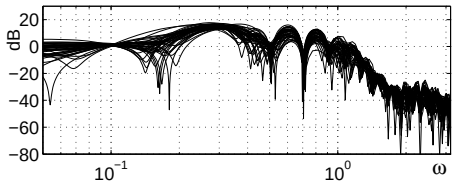

Fig. 5. Uncertainty at the first iteration (FIR models).

From Figure 3 a large error between $M(z,\widehat{\lambda}_1)$ and the true data-generating system is apparent. Correspondingly, one would also expect the estimated uncertainty to be quite large.

When uncertainty is estimated within Box-Jenkins models, the results is completely unreliable: the uncertainty concentrates around a model far from the true system, this would have lead to a destabilizing controller in the next step of the iterative algorithm.

When instead FIR models are used, the uncertainty is correctly estimated (in fact, it is very scattered around the true system) so that it has been possible to perform the subsequent controller design step. In fact, the randomized algorithms have been applied with $\varepsilon = 0.1$ and $\delta = 0.01$ while the parameter set of feasible controllers $\Gamma = [0,1]$ has been sampled in $p = 30$ points. The resulting number q of models extracted according to the probability density $f_1^2(\vartheta)$ was 435.

The controller obtained at the first iteration is characterized by a detuning parameter $\overline{\gamma}_1$ equal to 0.06. Its small value indicates a conservative choice which is justified by the high level of uncertainty. The step-response of the obtained closed-loop system is depicted in Figure 8.

Carrying on the iterative procedure the identified nominal model becomes a more and more accurate description of the real plant (Figure 6), and, correspondingly, uncertainty tends to concentrate around the true system (see e.g. Figure 7). This leads to the selection of $\overline{\gamma}_i$'s as indicated in Figure 9.

As it appears the control performance rapidly improves through iterations, preserving always the robust stability (see Figure 8). Figure 10 represents the empirical average cost $\widehat{c}(\overline{\gamma}_i)$ through iterations.

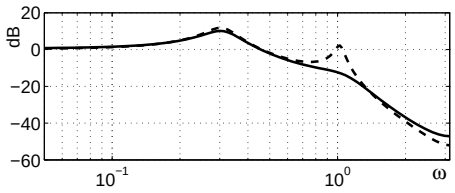

Fig. 6. Estimated nominal models at iterations $i = 2$ ("—") and $i = 3$ ("- -").

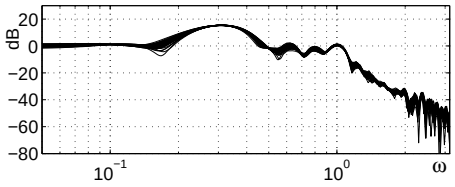

Fig. 7. Uncertainty at the third iteration ($i = 3$).

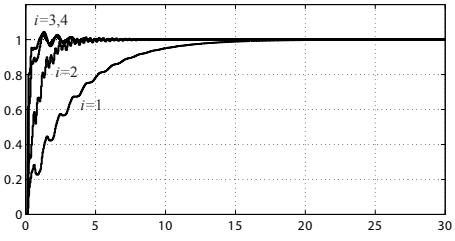

Fig. 8. Step response of the closed loop – $i = 1,\ldots,4$.

6. ACKNOWLEDGEMENTS

Paper supported by the MIUR National Research Project "Innovative Techniques for Identification and Adaptive Control of Industrial Systems", and partially supported by CNR-IEIIT

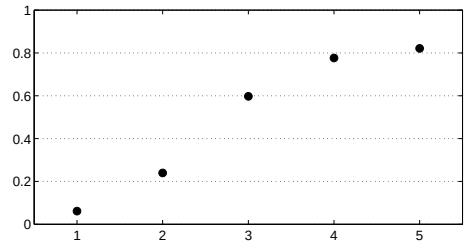

Fig. 9. $\overline{\gamma}_i$ at each iteration.

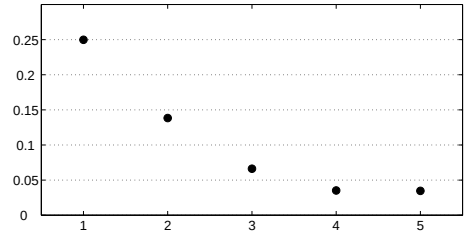

Fig. 10. $\widehat{c}(\overline{\gamma}_i)$ plotted for each iterations.

REFERENCES

Anderson, B.D.O., X. Bombois, M. Gevers and C. Kulcsàr (1998). Caution in iterative modeling and control design. *IFAC Workshop on Adaptive Control and Signal Processing* (Glasgow, UK), 13–19.

Bitmead, R.R., M.Gevers and A.G. Partanen (1997). Introducing caution in iterative controller design. *11th IFAC Symposium on System Identification* (Fukuoka, Japan), 1701–1706.

Bittanti, S., M.C. Campi and S. Garatti (2002). An iterative controller design scheme based on average robust control. *15th World IFAC Congress* (Barcelona, Spain).

Campi, M.C. and M. Prandini (2003). Randomized algorithms for the synthesis of cautious adaptive controllers. *Systems & Control Letters* **49**, 21–36.

De Callafon, R.A. and P.M.J. Van den Hof (1997). Suboptimal feedback control by a scheme of iterative identification and control design. *Mathematical Modelling of System* **3**, 77–101.

Garatti, S., M.C. Campi and S. Bittanti (2003). On the assessment of the model quality through the asymptotic theory for an instrumental variable setting. *42nd IEEE Conference on Decision and Control* (Hawaii, USA), 6015-6020.

Garatti, S., M.C. Campi and S. Bittanti (2004). Assessing the quality of identified models trough the asymptotic theory - when is the result reliable? *Automatica* – to appear (August 2004).

Gevers, M. (2000). A decade of progress in iterative control design: from theory to practice. *IFAC Symposium on Advanced Control of Chemical Processes* (Pisa, Italy), 677–694.

Landau, I.D., D. Rey, A. Karimi, A. Voda and A. Franco (1995). A flexible transmission system as a benchmark for robust digital control. *European Journal of Control - special issue*.

Lee, W.S., B.D.O. Anderson, I.M.Y. Mareels and R.L. Kosut (1995). On some key issues in the windsurfer approach to adaptive robust control. *Automatica* **31**, 1619–1636.

Ljung, L. (1999). *System Identification: Theory for the User*. Prentice-Hall. Upper Saddle River, NJ.

Van den Hof, P.M.J. and R.J.P. Schrama (1995). Identification and control - closed-loop issues. *Automatica* **31**, 1751–1770.

Vidyasagar, M. (2001). Randomized algorithms for robust controller synthesis using statistical learning theory. *Automatica* **37**, 1515–1528.